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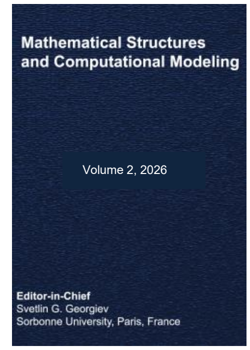

Spectral Analysis of Arithmetic Functions

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ABSTRACT

This paper explores the normalization of Fourier series kernels associated with elementary arithmetic functions. By analyzing key functions such as divisor related functions, the prime gap function and the inverse prime counting function. With these results, a framework for their spectral decomposition in terms of complex exponential functions was developed. Through contour integration techniques and normalization of the Dirac delta function, new methods can be derived from the analytic structure of these functions. These findings contribute to the broader understanding of arithmetic functions and their link to Fourier series.

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1. Introduction

Arithmetic functions occupy a central position in analytic number theory, serving as a bridge between the multiplicative structure of integers and the analytic properties of complex functions. Classical examples include the divisor function [5], [3] $d(n)$, the generalized divisor functions $d_k(n)$, the sum-of-divisors function $\sigma_a(n)$, and prime-related functions such as the prime counting function $\pi(x)$ and the prime gap sequence $g_n = p_{n+1} - p_n$. The study of such functions has historically relied on analytic techniques involving Dirichlet series [10], generating functions, and complex analysis [8].

The modern analytic approach to arithmetic functions originates from the work of Dirichlet, Riemann [6], and later Landau and Hardy [2]. Dirichlet introduced Dirichlet series as generating objects encoding arithmetic information through analytic functions of a complex variable. In particular, the Riemann zeta function [6]

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

and its Euler product representation establish a deep connection between analytic structures and the distribution of prime numbers. Riemann's seminal 1859 memoir demonstrated that the analytic continuation and zero distribution of $\zeta(s)$ govern the asymptotic behavior of the prime counting function $\pi(x)$.

Many arithmetic functions admit elegant representations through Dirichlet series involving the zeta function. For instance, the divisor function satisfies

$$\sum_{n=1}^{\infty} \frac{d(n)}{n^s} = \zeta^2(s),$$

while the generalized divisor functions satisfy

$$\sum_{n=1}^{\infty} \frac{d_k(n)}{n^s} = \zeta^{\kappa}(s).$$

Similarly, the sum-of-divisors functions satisfy

$$\sum_{n=1}^{\infty} \frac{\sigma_a(n)}{n^s} = \zeta(s)\zeta(s-a).$$

These relations illustrate how multiplicative arithmetic information is encoded within analytic objects, allowing the use of complex analytic methods such as contour integration, Mellin transforms [11], and residue calculus.

Another classical analytic tool is the Lambert series [7], which provides generating functions for divisor-type arithmetic functions. Lambert series appear naturally in the study of partition theory, modular forms, and multiplicative convolution identities. Their analytic properties have been extensively studied in the context of both analytic number theory and q-series.

Fourier analysis [9] offers an alternative viewpoint for studying arithmetic structures. By expressing arithmetic constraints through exponential kernels, one can represent discrete objects using continuous analytic integrals. Such ideas appear in the study of the Poisson summation formula, explicit formulas in prime number theory, and spectral interpretations of arithmetic functions. In particular, Fourier-type representations of integer indicator functions provide a mechanism for encoding divisibility constraints through oscillatory exponential sums.

The distribution of prime numbers constitutes one of the central topics of analytic number theory. The Prime Number Theorem establishes that

$$\pi(x) \sim \frac{x}{\log x},$$

describing the asymptotic density of primes among the integers. Numerous refinements and related quantities have been studied, including prime gaps, inverse prime counting functions, and explicit formulas involving the zeros of the Riemann zeta function. These connections highlight the deep interplay between analytic structures and the multiplicative behavior of integers.

In recent years, renewed interest has emerged in representing arithmetic functions through spectral or transform-based frameworks. Such approaches attempt to interpret arithmetic objects as analytically invertible structures whose behavior can be analyzed using tools from harmonic analysis and complex analysis. In this setting, generating functions, Mellin transforms, and contour integrals provide a natural language for describing arithmetic phenomena.

The aim of the present work is to develop a spectral framework for arithmetic functions based on normalized Fourier kernels and complex analytic techniques. By representing integer constraints through Fourier-type integrals, divisor-related functions can be expressed as spectral sums of complex exponentials. These constructions naturally lead to Mellin-type representations and contour integral formulas for various arithmetic functions.

The framework is then extended to prime-related quantities. Generating functions for the prime gap sequence and the prime counting function are introduced and analyzed using inverse transform methods and residue calculus. By combining these analytic constructions with asymptotic results derived from the Prime Number Theorem, explicit approximations for the inverse prime counting function and prime gaps are obtained. Notably, compact formulas involving the Lambert W function [4] arise naturally within this analytic setting.

Overall, this work aims to unify Fourier-analytic normalization with classical tools of analytic number theory. The resulting framework provides new integral representations and asymptotic expansions for divisor functions and prime-related quantities, illuminating structural connections between arithmetic functions, spectral decompositions, and their generating mechanisms.

2. Divisor Functions

This section develops a spectral and analytic construction of divisor functions. By expressing arithmetic constraints through normalized Fourier kernels and contour integrals, the divisor indicator function is written as an infinite exponential sum. This formulation allows divisor functions to be derived via complex-analytic methods, including residue calculus and inverse transforms. Define the integer indicator function:

$$1_{\mathbb{Z}}(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Z}, \\ 0, & \text{otherwise.} \end{cases}$$

for all $x \in R$. It admits the Fourier-type representation:

$$1_{\mathbb{Z}}(x) = \lim_{T \to \infty} \frac{1}{4\pi T} \int_{-2\pi T}^{2\pi T} \frac{e^{itx}}{e^{it} - 1} \, dt, \tag{2.0.1}$$

which, under the conformal change of variables $it = 2T\log(z)$, becomes the contour integral:

$$1_{\mathbb{Z}}(x) = \lim_{T \to \infty} \frac{1}{2\pi i} \oint_{|z|=1} \frac{z^{2Tx}}{z^{2T} - 1} \frac{dz}{z}. \tag{2.0.2}$$

By the residue theorem, this yields the classical exponential sum:

$$1_{z}(x) = \lim_{T \to \infty} \frac{1}{2T} \sum_{n=-T}^{T} e^{2\pi i n x}. \tag{2.0.3}$$

Consequently, the divisor indicator function satisfies:

$$1_{k|m} = 1_{\mathbb{Z}}(m/k) = \lim_{T\to\infty} \frac{1}{2T} \sum_{n=-T}^{T} e^{2\pi i n m/k} \quad m, k \in \mathbb{Z}, k \neq 0, \tag{2.0.4}$$

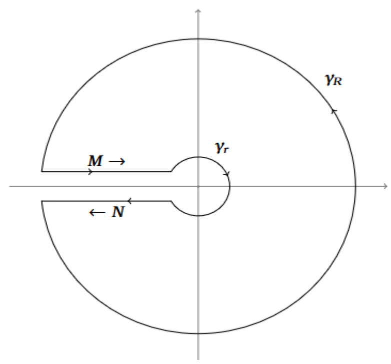
where:

$$1_{m|n} = \begin{cases} 1, & \text{if m|n,} \\ 0, & \text{otherwise,} \end{cases}$$

is the divisor indicator function. For subsequent developments we use the contour identity:

$$\frac{1}{2\pi i} \oint_C z^n \, (log(z))^k dz = \frac{d^k}{ds^k} \frac{sin(\pi(s+n))}{\pi(s+n)} |_{s=0}, \tag{2.0.5}$$

where curve C is a keyhole contour centered at the origin with argument $-\pi \leq arg(z) \leq \pi$, the sketch is the following:



Where it is centered to zero with argument $-\pi \leq arg(z) \leq \pi$, with the small radius $\epsilon > 0$ really small and the big radius equal to one $R = 1$. The generating function of the divisor function $d(n)$ admits the Mellin-type representation:

$$\sum_{n=1}^{\infty} d(n) z^{-n} = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \frac{\Gamma(s)}{(log(z))^s} \zeta^2(s) ds. \tag{2.0.6}$$

Evaluating the integral by residues and using $\zeta(-k) = \frac{B_{k+1}}{k+1}$ [6], we obtain:

$$\sum_{n=1}^{\infty} d(n) z^{-n} = \frac{-\gamma - log(log(z))}{log(z)} + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{B_{k+1}^2}{(k+1)^2} (log(z))^k, \tag{2.0.7}$$

where γ denotes the Euler–Mascheroni constant. Applying the inverse z-transform and the contour identity above 2.0.5 yields:

$$d(n) = log(n) + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{B_{k+1}^2}{(k+1)^2} \frac{d^k}{ds^k} \frac{sin(\pi(s+n))}{\pi(s+n)} |_{s=0}, \tag{2.0.8}$$

where B_k denotes the Bernoulli numbers. More generally,

$$\zeta^k(s) = \sum_{n=1}^{\infty} \frac{d_{k-1}(n)}{n^s}, \tag{2.0.9}$$

where:

$$d_{k-1}(n) = \sum_{a_1 a_2 \dots a_k = n} 1 \tag{2.0.10}$$

and $d_1(n) = d(n)$. Inversion of the zeta function gives:

$$1_{n\geq 1} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n^{\sigma+it} \zeta(\sigma + it) dt. \tag{2.0.11}$$

Using the expansions:

$$\zeta(s) = \sum_{k=0}^{\infty} \frac{(log(\zeta(s)))^k}{k!}, \tag{2.0.12}$$

and:

$$log(\zeta(s)) = \sum_{n=2}^{\infty} \frac{\Lambda(n)}{log(n)} \frac{1}{n^s}, \tag{2.0.13}$$

one obtains:

$$1_{n\geq 1} = 1,\ n = 1, \tag{2.0.14}$$

$$1_{n\geq 1} = \frac{\Lambda(n)}{log(n)} + \frac{1}{2!} \sum_{ab=n} \frac{\Lambda(a)\Lambda(b)}{log(a)log(b)} + \frac{1}{3!} \sum_{abc=n} \frac{\Lambda(a)\Lambda(b)\Lambda(c)}{log(a)log(b)log(c)} + \cdots,\ n \geq 2, \tag{2.0.15}$$

where $\Lambda(n)$ is the Von Mangoldt function defined by [1]:

$$\Lambda(n) = \lim_{T\to\infty} -\frac{\pi}{T} \sum_{\substack{\zeta(\rho)=0 \\ -T\leq\Im(\rho)\leq T \\ 0<\Re(\rho)<1}} n^{\rho}, n \in \mathbb{R}, n > 1. \tag{2.0.16}$$

This expansion generalizes to:

$$d_k(n) = 1,\ n = 1, \tag{2.0.17}$$

$$d_{k-1}(n) = \frac{\Lambda(n)}{log(\sqrt[k]{n})} + \frac{1}{2!} \sum_{ab=n} \frac{\Lambda(a)\Lambda(b)}{log(\sqrt[k]{a})log(\sqrt[k]{b})} + \frac{1}{3!} \sum_{abc=n} \frac{\Lambda(a)\Lambda(b)\Lambda(c)}{log(\sqrt[k]{a})log(\sqrt[k]{b})log(\sqrt[k]{c})} + \cdots,\ n \geq 2. \tag{2.0.18}$$

A generalized version of divisor functions is defined by the sum:

$$\sigma_a(n) = \sum_{d|n} d^a. \tag{2.0.19}$$

The Dirichlet series associated with $\sigma_a(n)$ is given by [5]:

$$D(\sigma_a, s) = \sum_{n=1}^{\infty} \frac{\sigma_a(n)}{n^s} = \zeta(s)\zeta(s-a),\ \Re(s) > 1 + max\{\Re(a), 0\}. \tag{2.0.20}$$

Consider now the contour integral representation:

$$\sigma_a(n) = n^{a/2} \oint_C z^{n-1} Q(z, a) \frac{dz}{2\pi i}, \tag{2.0.21}$$

where the function $Q(z,a)$ is defined by the generating series:

$$Q(z,a)=\sum_{n=1}^{\infty}\sum_{d|n}d^{a/2}\frac{d^{a/2}}{n^{a/2}}z^{-n}. \tag{2.0.22}$$

This function admits the Mellin integral representation:

$$Q(z,a)=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{\Gamma(s)}{(log(z))^{s}}\zeta\left(s-\frac{a}{2}\right)\zeta\left(s+\frac{a}{2}\right)ds. \tag{2.0.23}$$

Evaluating the integral using the residue theorem yields:

$$Q(z,a)=\zeta(a+1)\frac{\Gamma\left(\frac{a}{2}+1\right)}{(log(z))^{a/2+1}}+\zeta(1-a)\frac{\Gamma\left(-\frac{a}{2}+1\right)}{(log(z))^{-a/2+1}}+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\zeta\left(-k-\frac{a}{2}\right)\zeta\left(-k+\frac{a}{2}\right)log^{k}(z). \tag{2.0.24}$$

Substituting this result into 2.0.21 (together with 2.0.5) gives the following expression for the divisor power function:

$$\sigma_a(n)=\zeta(a+1)n^{a}+\zeta(1-a)+n^{a/2}\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\zeta\left(-k-\frac{a}{2}\right)\zeta\left(-k+\frac{a}{2}\right)\frac{d^{k}}{ds^{k}}\frac{sin\big(\pi(s+n)\big)}{\pi(s+n)}|_{s=0}. \tag{2.0.25}$$

An alternative construction uses the Lambert series:

$$L(q)=\sum_{n=1}^{\infty}b_{n}q^{-n}=\sum_{n=1}^{\infty}a_{n}\frac{1}{1-q^{n}}, \tag{2.0.26}$$

with:

$$b_n=\sum_{d|n}a_d. \tag{2.0.27}$$

The coefficients satisfy the inversion formula:

$$\mathrm{b_x}=\lim_{\mathrm{T}\to\infty}\frac{1}{2\mathrm{iT}}\int_{\sigma-iT}^{\sigma+iT}e^{sx}\ L(e^{s})ds, \tag{2.0.28}$$

with convergence estimate:

$$\left|\frac{1}{2iT}\int_{\sigma-iT}^{\sigma+iT}e^{sx}L(e^{s})ds\right|\leq\frac{1}{2T}\int_{-T}^{T}e^{\sigma x}\left|L(e^{\sigma+it})\right|dt\leq e^{\sigma x}\sum_{n=1}^{\infty}|b_n|e^{-\sigma n},\sigma>0. \tag{2.0.29}$$

Equivalently, by the residue theorem under the mapping $s=\frac{T}{\pi}log(z)$:

$$\mathrm{b_x}=\lim_{\mathrm{T}\to\infty}\frac{\pi}{\mathrm{T}}\sum_{\substack{|L(\rho_{\mathrm{i}})|=+\infty\\ -\mathrm{T}\leq\arg(\rho_{\mathrm{i}})\leq\mathrm{T}}}\mathrm{Res}(\mathrm{z^{x-1}L(z)};\rho_{\mathrm{i}}) \tag{2.0.30}$$

where p_i are the poles of the Lambert series.

3 Prime Generating and Prime gap Function

This section extends the spectral framework to prime-related quantities by introducing generating functions for the prime gap sequence and the prime counting function. Using contour integration and inverse transform techniques, the prime gap function is represented through complex-analytic constructions analogous to those developed earlier for divisor functions. The section also derives formulas for the inverse prime counting function via residue calculus, establishing a structural link between generating functions, Dirichlet series, and prime distribution. These results prepare the ground for the asymptotic approximations developed in the following section. The generating function of the prime gap sequence g_n is defined by:

$$G(z) = \sum_{k=0}^{\infty} g_k z^{-k} = \sum_{k=0}^{\infty} z^{-\pi(k)} \ , |z| > 1 \ , \tag{3.0.1}$$

where $\pi(k)$ denotes the prime counting function and $g_0 = p_1 = 2$. Equivalently, $G(z)$ can be expressed by the integral:

$$G(z) = \int_0^{\infty} z^{-\pi(t)}\, dt \ , |z| > 1. \tag{3.0.2}$$

The prime gap function admits the inversion formula:

$$\mathrm{g_x} = \lim_{\mathrm{T}\to\infty} \frac{1}{2\mathrm{iT}} \int_{\sigma-\mathrm{iT}}^{\sigma+\mathrm{iT}} \mathrm{e^{sx}G(e^s)ds}, \tag{3.0.3}$$

with convergence estimate is given by the inequalities:

$$\left|\frac{1}{2iT}\int_{\sigma-iT}^{\sigma+iT} e^{sx}\, G(e^s) ds\right| \le \frac{1}{2T}\int_{-T}^{T} e^{\sigma x} \left|G(e^{\sigma+it})\right| dt \le e^{\sigma x} G(e^{\sigma}), \sigma > 0. \tag{3.0.4}$$

By the residue theorem under the conformal mapping $s = \frac{T}{\pi} log\, z$, this becomes:

$$g_x = \lim_{T\to\infty} \frac{\pi}{T} \sum_{\substack{|G(\rho_i)|=+\infty \\ -T\le arg(\rho_i)\le T}} Res(z^{x-1} G(z); \rho_i), \tag{3.0.5}$$

where p_i are the poles of the prime gap generating function $G(z)$. Using the inverse Z-transform and identity 2.0.5, we obtain:

$$g_n = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (\pi(m))^k \frac{d^k}{ds^k} \frac{sin\big(\pi(s+n)\big)}{\pi(s+n)} |_{s=0}. \tag{3.0.6}$$

The inverse prime counting function is given by the contour integral:

$$\pi^{-1}(x) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma-i\infty} \frac{e^{s(x-1)}}{s} G(e^s) ds, \tag{3.0.7}$$

and, by residue calculus:

$$\pi^{-1}(\mathrm{x}) = \mathrm{Res}(\frac{e^{s(x-1)}}{s} G(e^s); 0) + \sum_{\substack{|G(\rho_i)|=+\infty \\ \rho_{i\neq 1}}} Res(\frac{e^{s(x-1)}}{s} G(e^s); log(\rho_i)) \tag{3.0.8}$$

4. Asymptotic Approximations

This section develops asymptotic formulas for the prime gap function and the inverse prime counting function using Dirichlet series techniques and the Prime Number Theorem. By approximating the associated Dirichlet series and evaluating the resulting contour integrals, explicit asymptotic expansions are obtained. In particular, closed-form approximations involving the Lambert W function are derived, leading to logarithmic-type growth estimates for prime gaps. These results provide analytic insight into the large-scale behavior of prime-related functions within the spectral framework established earlier. The asymptotic approximations of the prime gap function and the inverse prime counting function, are derived by considering the Dirichlet series:

$$D(g,s)=\sum_{n=1}^{\infty}\frac{g_n}{n^s}=\sum_{n=2}^{\infty}\frac{1}{\pi(n)^s}, \tag{4.0.1}$$

using the asymptotic approximation of the prime counting function $\pi(x)\sim\frac{x}{logx}$, according to the prime number theorem [6], the Dirichlet series for the prime gap function gets the form:

$$D(g,s)\sim\sum_{n=1}^{\infty}\frac{(log\,n)^s}{n^s}\ ,\Re(s)>1, \tag{4.0.2}$$

the Dirichlet series for the prime gap function can be expressed by the integral:

$$D(g,s)=\int_2^{\infty}\frac{1}{(\pi(x))^s}dx, \tag{4.0.3}$$

equivalently, using the asymptotic approximation of the prime counting function:

$$D(g,s)\sim\int_1^{\infty}\frac{(log\,x)^s}{x^s}dx, \tag{4.0.4}$$

by evaluating the integral gives the result:

$$D(g,s)\sim\frac{\Gamma(s+1)}{(s-1)^{s+1}}. \tag{4.0.5}$$

The inverse prime counting function admits the Mellin inversion formula:

$$\pi^{-1}(x)=2+\int_{2-i\infty}^{2+i\infty}\frac{(x-1)^s}{s}D(g,s)\frac{ds}{2\pi i}\ ,x>1. \tag{4.0.6}$$

Substituting the asymptotic expression of $D(g,s)$ gives:

$$\pi^{-1}(x)\sim 2+(x-1)log(x-1)-(x-1)\sum_{k=1}^{\infty}\frac{k^{k-1}}{k!}\frac{1}{(x-1)^k}\ ,x>1+e. \tag{4.0.7}$$

The contour integral can be evaluated in closed form using the Lambert W function [4]:

$$\int_{2-i\infty}^{2+i\infty}\frac{(x-1)^s}{s}\frac{\Gamma(s+1)}{(s-1)^{s+1}}\frac{ds}{2\pi i}=-(x-1)W_{-1}\left(\frac{1}{1-x}\right), \tag{4.0.8}$$

leading to the approximation:

$$\pi^{-1}(x)\sim 2-(x-1)W_{-1}\left(\frac{1}{1-x}\right)\ ,x>1+e, \tag{4.0.9}$$

the corresponding graph of the approximation above, is given below:

Using the average order theorem [12], $g_n\sim\frac{d\pi^{-1}(x+1)}{dx}|_{x=n}$:

$$g_n\sim log(n)+1-\sum_{k=1}^{\infty}\frac{k^{k-1}(1-k)}{k!}\frac{1}{n^k}\ ,n>e, \tag{4.0.10}$$

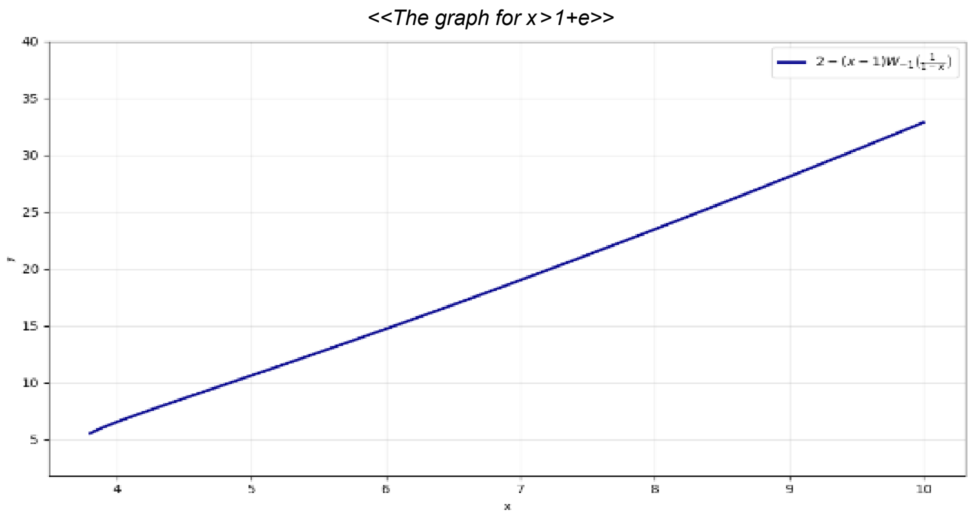


Figure 1: Inverse prime counting function $\pi^{-1}(x)$ approximation graph.

equivalently, using the Lambert W function and its derivative:

$$g_n \sim -\frac{(W_{-1}(-\frac{1}{n}))^2}{1 + W_{-1}\left(-\frac{1}{n}\right)}\ ,n > e, \tag{4.0.11}$$

the corresponding graph of the approximation above, is given by:

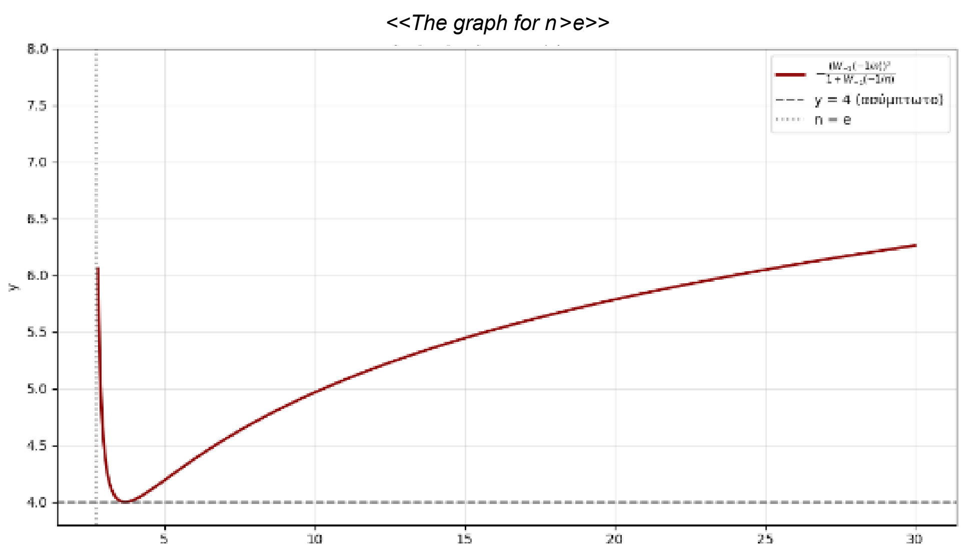


Figure 2: Prime gap function g_n approximation graph.

5. Conclusions and Final Remarks

In this paper, a unified spectral framework for the study of arithmetic functions is developed by combining Fourier kernel normalization, contour integration, and Dirichlet series methods. These constructions linked divisor functions to the Lambert series and illustrating the flexibility of the spectral approach.

The framework was then extended to prime-related quantities. By introducing generating functions for the prime gap sequence and the prime counting function, and applying inverse transform and residue techniques, explicit integral representations and summation formulas were obtained. Through asymptotic analysis grounded in the Prime Number Theorem, approximations for the inverse prime counting function and prime gaps was derived, including compact expressions involving the Lambert W function. These results are consistent with the expected logarithmic growth behavior of primes and their gaps.

Overall, the paper demonstrates that arithmetic functions can be treated as analytically invertible spectral objects. The unification of Fourier-analytic normalization with classical tools of analytic number theory provides new integral representations and asymptotic expansions, clarifying structural relationships between divisor functions, prime distributions, and their generating mechanisms.

Future work may further investigate the analytic properties of the associated generating functions, explore refinements of the asymptotic expansions, and examine potential applications of the spectral framework to other arithmetic functions and related problems in analytic number theory.

Conflict of Interest

The authors report that they have no conflicts of interest to disclose.

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